

4. V – M – Lomené výrazy

0) Napište si obecné vzorce, které budete při úpravě lomených výrazů potřebovat:

$$a \cdot (b + c) = ab + ac$$

$$\frac{c \cdot a}{c \cdot b} = \frac{a}{b}$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$\frac{a}{b} : \frac{c}{d} = \frac{ad}{bc}$$

$$(a + b) \cdot (a - b) = a^2 - b^2$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$(x + a) \cdot (x + b) = x^2 + (a + b)x + ab$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

1) Rozšiřte zadaný výraz určeným výrazem, určete podmínky:

Čím: $(x+6)$, výraz: $\frac{4}{2-3x} = \frac{x+6}{x+6} \cdot \frac{4}{2-3x} = \frac{4x+24}{-3x^2-16x+12}; x \neq \frac{2}{3}; -6$

a) $-3, \frac{-5}{4x+7} = \frac{15}{-12x-21}; x \neq -\frac{7}{4}$

b) $4, \frac{2x-2}{5x} = \frac{8x-8}{20x}; x \neq 0$

c) $3x, \frac{7x}{4-x^2} = \frac{21x^2}{-3x^3+12x}; x \neq \pm 2; 0$

d) $-2x, \frac{x^2-2x+1}{6-x} = \frac{-2x^3+4x^2-2x}{x^2-12x}; x \neq 0; 6$

e) $\frac{x}{2}, \frac{2 \cdot (x+1)}{x^2-1} = \frac{x \cdot (x+1)}{\frac{x^3}{2} - \frac{x}{2}}; x \neq \pm 1; 0$

f) $\frac{3x}{4}, \frac{5x^2+3x-2}{3 \cdot (x+1)} = \frac{\frac{15x^3}{4} + \frac{9x^2}{4} - \frac{3x}{2}}{\frac{9x}{4} \cdot (x+1)}; x \neq -1; 0$

g) $(x-1), \frac{6x}{9-3x} = \frac{6x^2-6x}{-3x^2+12x-9}; x \neq 1; 3$

h) $(2-3x), \frac{1-x^2}{6x+10} = \frac{3x^3-2x^2-3x+2}{-18x^2-18x+20}; x \neq -\frac{5}{3}; \frac{2}{3}$

i) $\frac{2}{5}, \frac{\frac{10}{6}x^2-15}{5x} = \frac{\frac{3}{2}x^2-6}{2x}; x \neq 0$

j) $-\frac{3}{8}, \frac{\frac{16}{6}-4x^2}{12x+8} = \frac{\frac{3}{2}x^2-1}{-\frac{9}{2}x-3}; x \neq -\frac{2}{3}$

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2) Zkraťte zadaný výraz do co možná nejjednodušší podoby, určete podmínky:

$$\frac{x^2 - 169}{3x + 39} = \frac{(x+13)(x-13)}{3 \cdot (x+13)} = \frac{x-13}{3}; x \neq -13$$

a) $\frac{14}{21} = \frac{2}{3}$

b) $\frac{72}{32} = \frac{9}{4}$

c) $\frac{3x}{15x^2} = \frac{1}{5x}; x \neq 0$

d) $\frac{64x^5}{40x^3} = \frac{8x^2}{5}; x \neq 0$

e) $\frac{32x^2 - 40x}{8x} = 4x - 5; x \neq 0$

f) $\frac{-12x^2}{28x + 16x^2} = \frac{-3x}{7 + 4x}; x \neq -\frac{7}{4}; 0$

g) $\frac{x+8}{5x+40} = \frac{1}{5}; x \neq -8$

h) $\frac{8x-28}{2x-7} = 4; x \neq \frac{7}{2}$

i) $\frac{x^2 - 6x + 9}{x-3} = x - 3; x \neq 3$

j) $\frac{2x+3}{4x^2 + 12x + 9} = \frac{1}{2x+3}; x \neq -\frac{3}{2}$

k) $\frac{x^2 + 10x + 25}{x^2 - 25} = \frac{x+5}{x-5}; x \neq \pm 5$

l) $\frac{x^2 - 16}{x^2 - 8x + 16} = \frac{x+4}{x-4}; x \neq 4$

m) $\frac{7x+21}{2x^2 - 18} = \frac{7}{2x-6}; x \neq \pm 3$

n) $\frac{3x^2 - 12x + 12}{4x - 8} = \frac{3x-6}{4}; x \neq 2$

o) $\frac{x^2 - 4x - 5}{6x + 6} = \frac{x-5}{6}; x \neq -1$

p) $\frac{x^2 + 2x - 24}{x^2 - 16} = \frac{x+6}{x+4}; x \neq \pm 4$

3) Proveďte násobení či dělení výrazů, pokud to lze, výsledek zkraťte, určete podmínky:

$$\frac{x^2 - x - 12}{2x + 14} : \frac{x-4}{4x+28} = \frac{x^2 - x - 12}{2x + 14} \cdot \frac{4x + 28}{x-4} = \frac{(x-4)(x+3)}{2(x+7)} \cdot \frac{4 \cdot (x+7)}{x-4} = 2x + 6; x \neq -7; 4$$

a) $\frac{5}{7} \cdot \frac{14}{15} = \frac{2}{3}$

b) $\frac{9}{16} : \frac{3}{32} = 6$

c) $\frac{8x}{21} \cdot \frac{7x^2}{4x} = \frac{2x^2}{3}; x \neq 0$

d) $\frac{12x}{x^3} : \frac{5x}{2x^2} = \frac{24}{5x}; x \neq 0$

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$$e) \frac{x+4}{x-7} : \frac{x}{2x-14} = \frac{2x+8}{x}; x \neq 0; 7$$

$$f) \frac{5x}{6-3x} \cdot \frac{2-x}{5} = \frac{x}{3}; x \neq 2$$

$$g) \frac{x^2-14x+49}{7-2x} \cdot \frac{6x-21}{x-7} = 21-3x; x \neq \frac{7}{2}; 7$$

$$h) \frac{27x+9}{x^2-36} : \frac{1+3x}{x+6} = \frac{9}{x-6}; x \neq \pm 6; -\frac{1}{3}$$

$$i) \frac{x^2-4x-21}{2x^2-200} : \frac{x-7}{x^2-20x+100} = \frac{x^2-7x-30}{2x+20}; x \neq \pm 10; 7 \quad j) \frac{x-1}{x^2+3x-4} \cdot \frac{2x+8}{x^2} = \frac{2}{x^2}; x \neq -4; 1; 0$$

4) Provedte sčítání či odčítání výrazů, určete podmínky:

$$\begin{aligned} \frac{x^2-3x+2}{2x-9} + \frac{6x-4}{4x-18} &= \frac{2}{2} \cdot \frac{x^2-3x+2}{2x-9} + \frac{6x-4}{4x-18} = \\ &= \frac{2x^2-6x+4}{4x-18} + \frac{6x-4}{4x-18} = \frac{2x^2}{4x-18} = \frac{x^2}{2x-9}; x \neq \frac{9}{2} \end{aligned}$$

$$a) \frac{5}{3-2x} + \frac{x-2}{3-2x} = \frac{x+3}{-2x+3}; x \neq \frac{3}{2}$$

$$b) \frac{x^2-3x+4}{12x+1} - \frac{x^2+4}{12x+1} = \frac{-3x}{12x+1}; x \neq -\frac{1}{12}$$

$$c) \frac{4-x}{x-8} - \frac{3+4x}{3x-24} = \frac{-7x+9}{3x-24}; x \neq 8$$

$$d) \frac{2x+3}{6x+18} + \frac{-1}{2x+6} = \frac{2x}{6x+18}; x \neq -3$$

$$e) \frac{x+4}{x^2-11x} - \frac{1}{x-11} = \frac{4}{x^2-11}; x \neq 0; 11$$

$$f) \frac{x^2-2x+1}{5x^2} + \frac{2}{5x} = \frac{x^2+1}{5x^2}; x \neq 0$$

$$g) \frac{2x+15}{x^2-12x+36} - \frac{2}{x-6} = \frac{27}{x^2-12x+36}; x \neq 6$$

$$h) \frac{3}{4x-8} + \frac{-11x-24}{16x^2-64} = \frac{x}{16x^2-64}; x \neq \pm 2$$

$$i) \frac{x}{x+4} + \frac{5x-x^2}{x^2-x-20} = 0; x \neq -4; 5$$

$$j) \frac{x^2-x}{x^2+2x-35} - \frac{2x+1}{2x+14} = \frac{7x+5}{2x^2+4x-70}; x \neq -7; 5$$

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5) Proveďte operace, zjednodušte výraz, určete podmínky:

$$\frac{x^2 - 5x - 6}{x^2 - 1} + \frac{6}{4x - 4} \cdot \frac{x + 2}{2} = \frac{(x - 6) \cdot (x + 1)}{(x - 1) \cdot (x + 1)} + \frac{6 \cdot (x + 2)}{8 \cdot (x - 1)} =$$

$$= \frac{8}{8} \cdot \frac{x - 6}{x - 1} + \frac{6 \cdot (x + 2)}{8 \cdot (x - 1)} = \frac{8x - 48}{8x - 8} + \frac{6x + 12}{8x - 8} = \frac{7x - 18}{4x - 4}; x \neq -1; 1$$

$$a) \frac{2}{4 + 3x} + \frac{x - 2}{16 - 9x^2} = \frac{6 - 5x}{16 - 9x^2}; x \neq \pm \frac{4}{3}$$

$$b) \frac{2x - 1}{x} \cdot \frac{2}{x - 3} - \frac{4x + 1}{2x^2 - 6x} = \frac{4x - 5}{2x^2 - 6x}; x \neq 0; 3$$

$$c) \frac{4 - 2x}{4x^2 - 16} : \frac{4x + 8}{-3} = \frac{3}{2(2x + 4)^2}; x \neq \pm 2$$

$$d) \frac{x^2 - 49}{6x + 42} \cdot \frac{5x - 35}{x^2 - 14x + 49} = \frac{5}{6}; x \neq \pm 7$$

$$e) \frac{2x}{x^2 - 6x} - \frac{1}{3x - 18} = \frac{5}{3x - 18}; x \neq 0; 6$$

$$f) \frac{-x + 8}{x^2 - 2x - 15} + \frac{2}{x + 3} = \frac{x - 2}{x^2 - 2x - 15}; x \neq -3; 5$$

$$g) \frac{3}{5} : \frac{6 - 4x}{x^2 - 64} - \frac{2}{16x - 24} = \frac{6x^2 - 379}{-40x + 60}; x \neq \pm 8; \frac{3}{2}$$

$$h) \frac{2x}{3 - x} \cdot \frac{1}{2x + 6} + \frac{2x - 3}{x^2 - 9} = \frac{1}{x + 3}; x \neq \pm 3$$

$$i) \frac{x^2 + 10x - 11}{6x + 6} : \frac{2x^2 - 22x}{3x} = \frac{(x - 1)(x + 11)}{4(x + 1)(x - 11)}; x \neq -1; 0; 11$$

$$j) \frac{25x^2 - 4}{50x^2 + 40x + 8} - \frac{4}{5x + 4} = \frac{25x^2 - 30x - 24}{50x^2 + 60x + 16}; x \neq -\frac{4}{5}; -\frac{2}{5}$$